## 4.1 A Formal Language for Predicate Logic

$$v \forall x(Gxy \leftrightarrow \exists yHy)$$

\*vi 
$$\exists x(Ax \rightarrow \forall xFxx)$$

\*vii 
$$\forall x \forall y (Fxy \rightarrow \forall z (Hxyz \& Jz))$$

\*viii 
$$\forall xFxx \leftrightarrow \forall x \forall yFxy$$

\*ix 
$$\sim \forall x \sim \exists z (Hz \vee Jx)$$

$$*x P \rightarrow Gab$$

\*xi Ga 
$$\rightarrow \forall x \sim (Ha \lor Fxx)$$

\*xii 
$$\sim$$
(P &  $\sim \exists xFx$ )

\*xiii 
$$\forall x(Fx) \& P$$

\*xiv 
$$\exists y(Fyyy \& P)$$

\*xv 
$$\forall$$
xyz(Fzx  $\leftrightarrow$  Hxyz)

## quantifier convention

*Comment.* When a wff contains an uninterrupted sequence of quantifiers of the same type, existential or universal, it is often convenient to omit repetitions of  $\exists$  or  $\forall$ .

Examples.

The expression

$$\forall xyz (Fxy \ \& \ Gyz \leftrightarrow Hzx)$$

will be read as shorthand for

$$\forall x \forall y \forall z (Fxy \ \& \ Gyz \leftrightarrow Hzx)$$

The expression

$$\exists xy \forall zw (Fxyz \& Gwx \rightarrow \sim Hzx)$$

is to be read as

$$\exists x \exists y \forall z \forall w (Fxyz \& Gwx \rightarrow \sim Hzx)$$

## open formula

Definition. An **OPEN FORMULA** is the result of replacing at least one occurrence of a name in a wff with a new variable (one not already occurring in the wff). They may also be obtained by removing the prefixed quantifier from a universal or existential wff.

55