

## 5.1 Primitive Rules of Proof

67

**\*Exercise 5.1.1** Pair wffs and their instances from the list of sentences below. Some formulas may appear in several pairs. Others may appear in none.

- i  $\forall xFax$
- ii  $\exists x(Fxa \ \& \ \forall yGyxa)$
- iii  $\exists xFax$
- iv  $Fab$
- v  $\exists y\forall xFyx$
- vi  $\exists zx(Fxz \ \& \ \forall yGyxa)$
- vii  $\forall xyFxy$
- viii  $\forall xFxa$
- ix  $\exists zx(Fxz \ \& \ \forall yGyxz)$
- x  $Fba \ \& \ \forall yGyba$

*Comment.* The primitive rules of proof for predicate logic include all the primitive rules from chapter 1. There are also introduction and elimination rules for the two quantifiers and for the identity symbol. Two of the new rules have conditions that must be met for the application of the rules to be correct.

**universal-elim**

Given a universally quantified sentence (at line  $m$ ), conclude any instance of it.

*Condition:* None.

*Annotation:*  $m \forall E$

*Assumption set:* same as line  $m$ .

*Also known as:* Universal Instantiation.

Examples.

(a)

1	(1)	$\forall xFx$	A
1	(2)	$Fa$	1 $\forall E$
1	(3)	$Fb$	1 $\forall E$