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$\forall$xFx $\rightarrow$ $\forall$xGx $\vdash$ Fm $\rightarrow$ $\exists$xGx

Model:
U: {m,a}
F: {m}
G: { }

Expansion:
The premise ($\forall$xFx $\rightarrow$ $\forall$xGx) expands to

```
Fm & Fa → Gm & Ga
 T  & F       F  & F
     F    →       F
            T
```

The conclusion (Fm $\rightarrow$ $\exists$xGx) expands to

```
Fm → Gm ∨ Ga
 T        F  ∨  F
 T  →       F
     F
```

The conclusion is false in this interpretation and the premise is true; hence, this interpretation is a countermodel for the given sequent.

Exercise 6.2 Construct countermodels and expansions to show the following sequents invalid.

*i $\forall$xFx $\rightarrow$ $\forall$x Gx $\vdash$ $\forall$x(Fx $\rightarrow$ Gx)
*ii $\exists$xFx $\rightarrow$ $\exists$xGx $\vdash$ $\forall$x(Fx $\rightarrow$ Gx)
*iii $\exists$xFx & $\exists$xGx $\vdash$ $\exists$x(Fx & Gx)
*iv $\exists$x(Fx $\vee$ Gx) $\vdash$ $\forall$xFx $\vee$ $\forall$xGx
*v $\exists$x(Fx $\rightarrow$ Gx) $\vdash$ $\exists$xFx $\rightarrow$ $\exists$xGx
*vi $\exists$x(Fx $\rightarrow$ Gx) $\vdash$ $\forall$xFx $\rightarrow$ $\forall$xGx
*vii $\forall$xFx $\leftrightarrow$ $\forall$xGx $\vdash$ $\forall$x(Fx $\leftrightarrow$ Gx)
*viii $\exists$xFx $\leftrightarrow$ $\exists$xGx $\vdash$ $\forall$x(Fx $\leftrightarrow$ Gx)